PRIMARY UNITS IN THIS GUIDE ARE METRIC (SI - THE INTERNATIONAL SYSTEM OF UNITS):

LENGTH	m	METER
MASS	g	GRAM
FORCE	mN	MILLINEWTON
TORQUE	mN·m	MILLINEWTON METER
INERTIA	g·m²	GRAM METER²

In this system, mass is always in kilograms or grams. Force, or weight is always in newtons or millinewtons.

Force (or weight) = Mass × Acceleration

F = ma

when a = 9.81 m/sec² (acceleration due to gravity), then F would be the weight in newtons.

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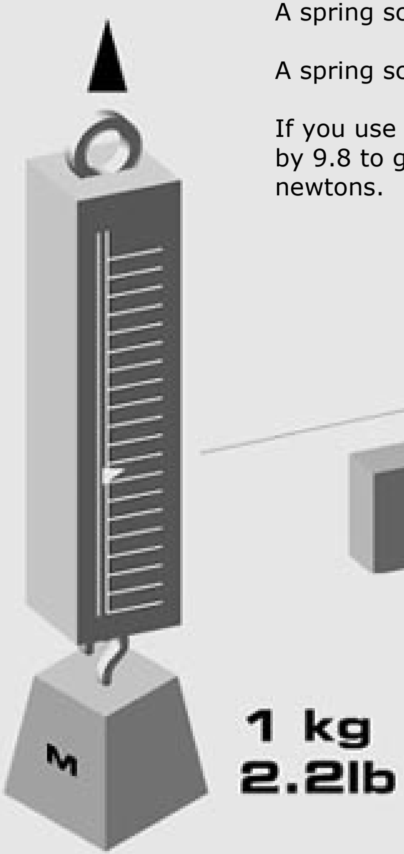
	GIVEN UNIT				UNITS USED IN THIS GUIDE (Metric SI)
LENGTH	1 inch	=	2.54 cm	=	2.54×10^{-2}m
FORCE	1 oz	=		=	278 mN
	1 lb	=	4.45 N	=	4,450 mN
	1 g·cm	=		=	9.8 mN
MASS	1 lb	=		=	454 g
	1 oz	=		=	28.4 g
	1 kg	=		=	1,000 g
	1 slug	=	14.6 kg	=	14,600 g
INERTIA	1 g·cm²	=		=	10^{-4} g·m²
	1 oz-in-sec²	=		=	7.06 g·m²
	1 slug ft²	=		=	0.29 g·m²
TORQUE	1 oz-in	=	72.01 g·cm	=	7.06 mNg·m
	1 lb-ft	=		=	1.356 × N·m
	1 g·cm	=		=	9.8×10^{-2} mN·m
			10.2 g·cm	=	1 mN·m
			141.6 oz-in	=	1 N·m

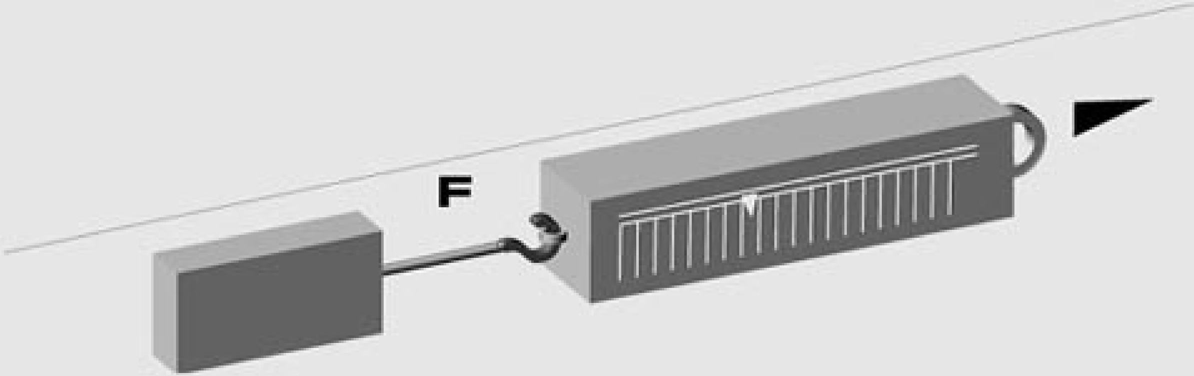
HOW TO MEASURE MASS OR FORCE



A spring scale reading of 1 kg means that you are measuring a mass of 1 kg.

A spring scale reading of 2.2 lb also is measuring a mass of 1 kg.

If you use that same spring scale to measure a force, the 1 kg reading must be multiplied by 9.8 to give a force of 9.8 newtons. The reading of 2.2 lb us a force and is equal to 9.8 newtons.



If the same scale is used to measure torque (T = FR) at one meter radius, the reading of

1 kilogram × 1 meter = 1 kgm

must be multiplied by 9.8 to give a torque of 9.8 newton meters (N·m).

1. TORQUE (mN·m) = FORCE (mN) × RADIUS (m)

Torque = FR

2. TORQUE REQUIRED TO ACCELERATE INERTIAL LOAD

$$T\ (\text{mN·m}) = J\,\alpha$$

J = Inertia in $g \cdot m^2$
α = Acceleration in radians/sec^2

EXAMPLE
If a rotor inertia plus load inertia = J = 2×10^3 $g \cdot m^2$, and the motor is to be accelerated at 6,000 radians per sec, what torque is required?

$T = J\,\alpha = 2 \times 10^3 \times 6000$
$T = 12$ mN·m

For stepper motors, α can be converted to radians/sec^2 from steps/sec^2.

$$\alpha\ (\text{radians/sec}) = \Delta v\ (\text{steps/sec}) \times \frac{2\pi}{\text{steps/rev}}$$

$$\textbf{TORQUE} = J\,\frac{\Delta v}{\Delta t} \times \frac{2\pi}{\text{steps/rev}}$$

EXAMPLE
For a 48-step per revolution motor accelerating from zero to steps/sec running rate v in Δt seconds.

$$\text{TORQUE} = J\,\frac{v}{\Delta t} \times \frac{\pi}{24}$$

If no acceleration time is provided, then a maximum 2-step lag can occur.

$$\Delta t\ (\text{sec}) = \frac{2\ (\text{steps})}{v\ (\text{steps/sec})}$$ giving the following equation:

$$\textbf{TORQUE} = J\,\frac{V^2}{2} \times \frac{2\pi}{\text{steps/rev}}$$

3. MOMENT OF INERTIA

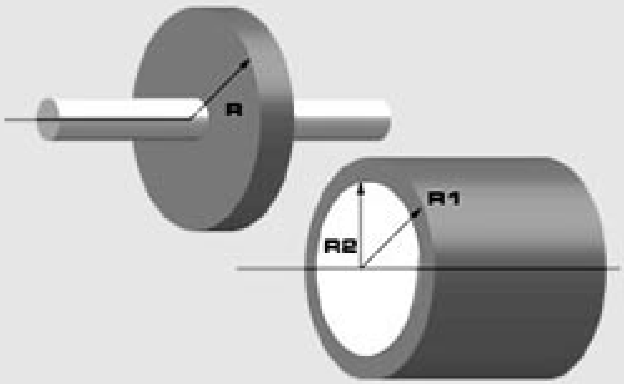
Disc or Shaft

$$J\ (g \cdot m^2) = \frac{MR^2}{2}$$

Cylinder

$$J\ (g \cdot m^2) = \frac{M}{2}(R^2{}_1 + R^2{}_2)$$

M = Mass in grams
R = Radius in meters

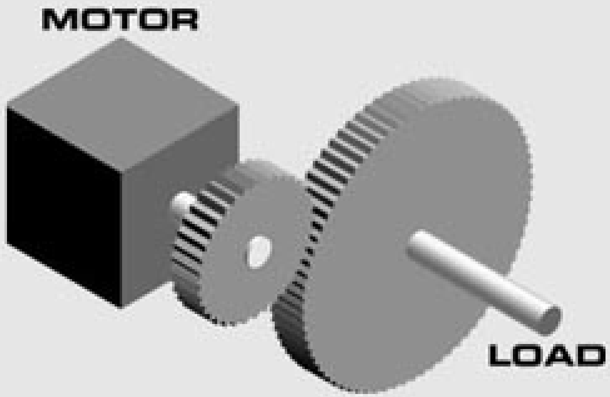


4. REFLECTED LOADS WHEN USING GEARS OR PULLEYS

$$\textbf{Torque required of motor} = \frac{\textbf{Load Torque}}{\textbf{GR}}$$

$$\text{gear or pulley ratio GR} = \frac{\text{motor shaft revolutions}}{\text{load shaft revolutions}}$$

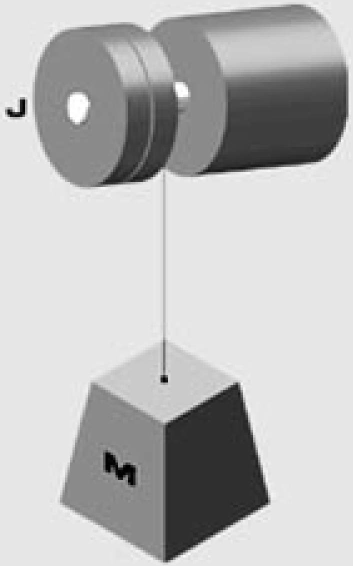
$$\textbf{Inertia reflected to motor} = \frac{\textbf{Load Inertia}}{\textbf{(GR)}^2}$$



5. EQUIVALENT INERTIAL LOAD

J eqv. (g·m²) = MR²

For a pulley and weight or a rack and pinion
M = Mass of load in grams
R = Radius of pulley in meters



6. TOTAL LOAD

Note: Be sure to include all load components.

J_T = Rotor Inertia + all J Loads

T_F = Frictional and Forces

Note: In the pulley example earlier, the total load would be:

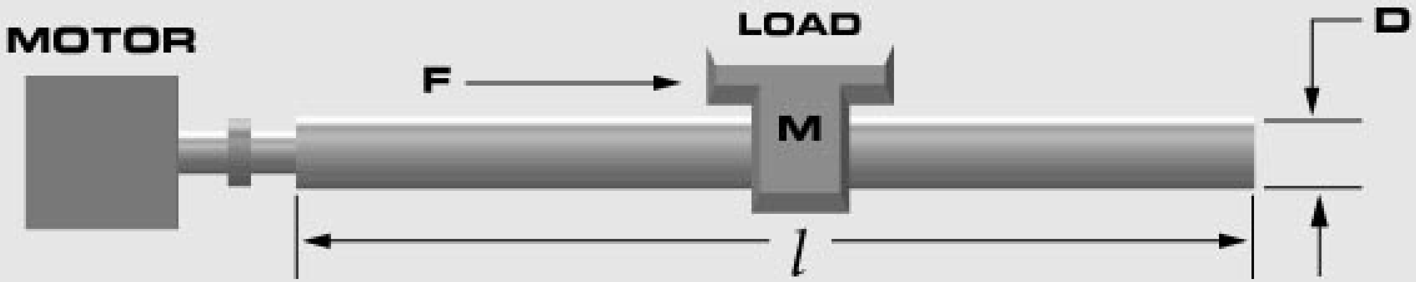
J_T = J rotor + J pulley + J eqv.

T_F = T frictional + Load Weight × Radius

Total T = $J_T \alpha + T_F$

Load weight = mass × 9.8 mN

7. AXIAL FORCE OF LEAD SCREW



$$F = \frac{2\Pi \times T}{L} \times \text{EFFICIENCY}$$

F (mN) when T = Torque in mN·m
L = Lead of screw in meters
F (oz) when T = Torque in oz-in
L = Lead of screw in inches
EFFECIENCY = from 0.9 for ballnut to 0.3 for Acme

Inertia of lead screw load

J = J rotor + J steel screw + J reflected

J steel screw = $D^{-4} \times l \times \pi/32 \times$ Density

Density for steel = 7.83×10^6 g/m³

then:

J reflected (g·m²) = $D^{-4}\ l \times 7.7 \times 10^5$

The reflected inertia of the load is:

J reflected (g·m²) = M (load) $L^2 \times 0.025$

Total Torque Load from lead screw (T) in mN·m

T = (J rotor + J screw + J reflected) α + T friction

8. MOTOR WATTS OUTPUT

Watts out = Torque output X speed in radians/sec

1 watt = 1 Nm/sec

For a given output torque (mN·m) and converting v (steps/sec) to radians/sec

$$\textbf{Watts out} = \textbf{Torque (mN·m)} \times v \frac{\text{(motor step angle)}}{57.3} \times 10^{-3}$$

If the speed is in RPM then:

$$\text{Watts out} = 1.05 \times 10^{-4} \times \text{torque (mN·m)} \times \text{RPM}$$

9. STEPS/REV TO RPM

$$\textbf{RPM} = \frac{v\ \text{(steps/sec)} \times 60}{\text{motor steps/rev}}$$