Theoretical developement on domestic 3D printing

About accuracy and sizing of delta printers

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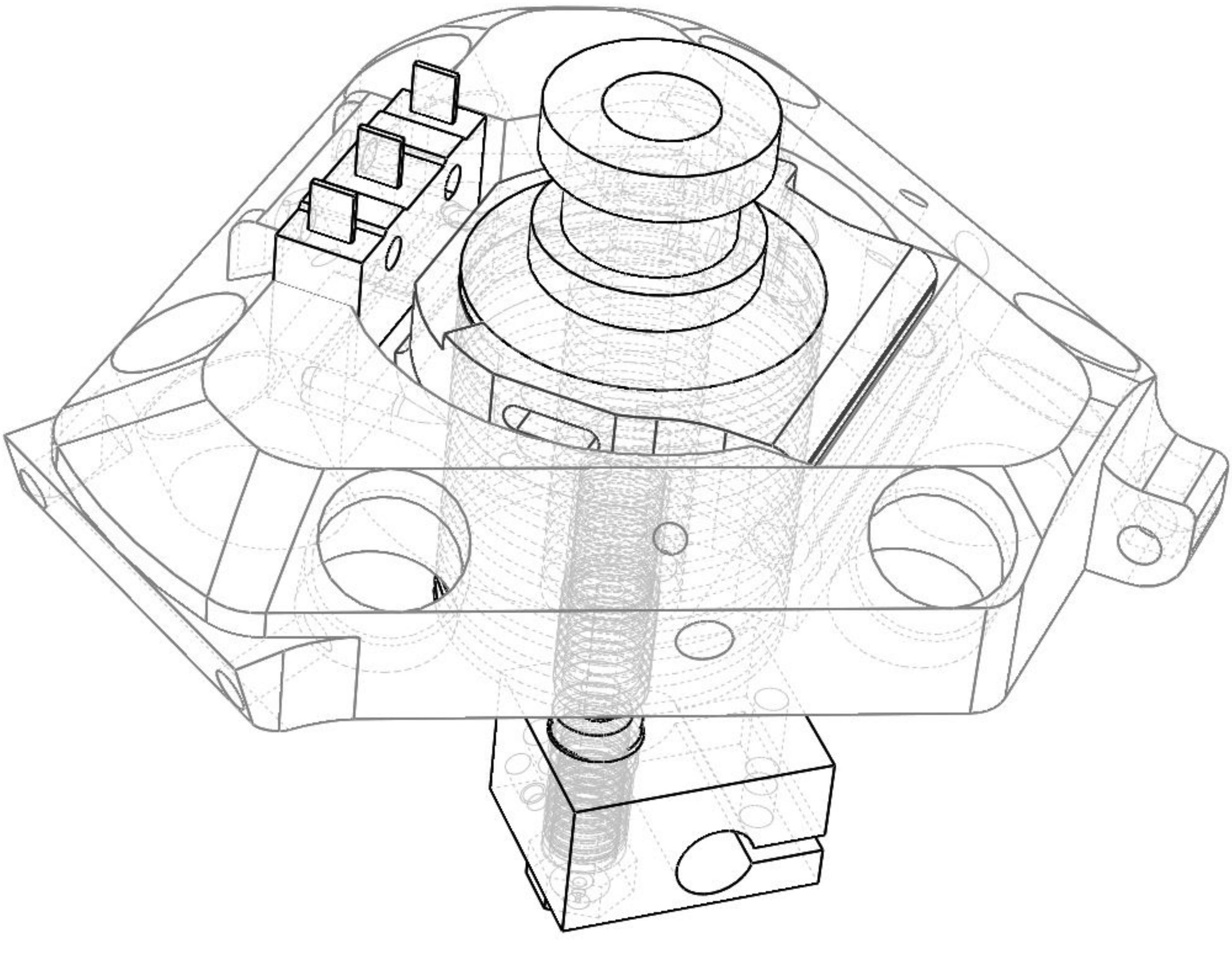

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Summary

If you are about to make a delta printer, or if you already did, probably you are familiar to this question: what's the correct size for this or that part of my printer? How do I decide my diagonal rod length? Is it important for accuracy? This text is going to give you some mathematical tools and concepts to solve those questions.

1. INTRODUCTION

This is what the text is going to develop: a compact effective mathematical development about the relationship between basic accuracy of printing, delta kinematics and how to calculate them easily. Of course, it is not mandatory to understand the mathematical stuff, you can just take the final equations and the explanation and work with it. Let's get started.

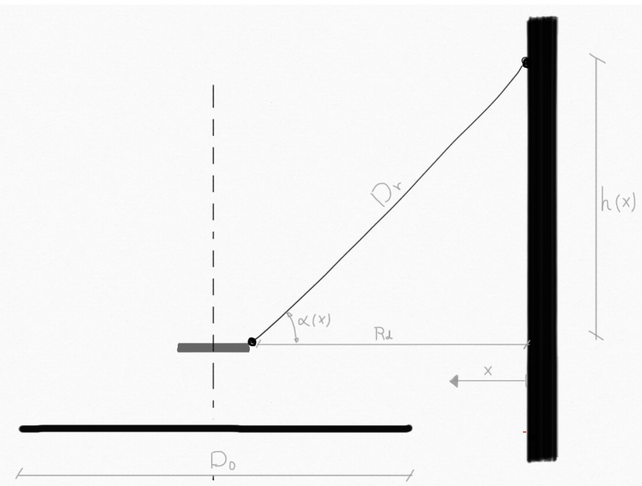

Figure 1: squeme of the important geometry dimensions for our calculations

Figure 1 show us the names of all different dimensions we are going to appeal. It should be clear that the derivation of nozzle displacement respect to the carriage displacement is going to give us direct information about the maximization or minimization of the minimum length step given by our nema motors. Let's proceed to that expression. First of all, the next expression describes the relationship between carriage position h and head position x (x is the horizontal distance between the point where the diagonal rod joins the head and the tower):

$$D_r^2 = h^2 + x^2 \tag{1}$$

After manipulation and derivation we get the next two expressions:

$$x = \sqrt{D_r^2 - h^2} \tag{2a}$$

$$\frac{\partial x}{\partial h} = \frac{-h}{\sqrt{D_r^2 - h^2}} \tag{2b}$$

Combining expressions 2a and 2b we can get the next one:

$$\frac{\partial x}{\partial h} = -\sqrt{\left(\frac{D_r}{x}\right)^2 - 1} \tag{3}$$

The magnitude of interest for us is just the rate of displacement of nozzle for every unit of carriage displacement, and for the numerical range of interest it is well described by the next relationship:

$$\rho(x) = -\frac{\partial x}{\partial h} = \sqrt{\left(\frac{D_r}{x}\right)^2 - 1} \tag{4}$$

Summarizing and clarifying for non mathematics connoisseur: A value of ρ superior to 1 means that the actual step size of the nozzle is higher than the step size of the carriage (first one is dependent of the position in the printing area, the second one is constant along the tower).

2. Accuracy on highlight points

Now is time to analyse the error on critical zones of the print area, so we will then base our sizing decision on those values. This places of the print area are the next:

$$
\begin{aligned}
x =& x_{min} = (R_d - D_0/2) && \text{(5a)}\\
x =& R_d && \text{(5b)}\\
x =& x_{max} = (R_d + D_0/2) && \text{(5c)}
\end{aligned}
$$

To get clearer equations we are going create the next non-dimensional variables:

$$
\begin{aligned}
a =& \frac{D_r}{R_d + D_0/2} && \text{(6a)}\\
b =& \frac{D_r}{R_d - D_0/2} && \text{(6b)}
\end{aligned}
$$

Applying the values 5 a, b and c on the expression 4, and the manipulating with the just mentioned non-dimenssional variables we get the next expressions:

$$
\begin{aligned}
\rho_{max} &= (x = x_{min}) = \sqrt{b^2 - 1} && \text{(7a)}\\
\rho_{center} &= (x = R_d) = \sqrt{\left(\frac{2ab}{a+b}\right)^2 - 1} && \text{(7b)}\\
\rho_{min} &= (x = x_{max}) = \sqrt{a^2 - 1} && \text{(7c)}
\end{aligned}
$$

The real practical minimum error is the one given on the center, because ρ_{min} is only given in the perpendicular direction of each tower.

3. Sizing your delta printer.

Now we have information of the step size on the critical points of our printing area is time to decide the size of the printer based on our own priorities fixing one or another variable.

3.1. Fixing minimum angle and maximum error.

So you have a given bed diameter and you want to make sure the controller does not overload when the angle goes to little and also want to make sure you have enough accuracy. [1]

$$
\begin{aligned}
\cos(\alpha_{min}) &= \frac{R_d + D_0/2}{D_r} = \frac{1}{a} && \text{(8a)}\\
\rho_{max} &= \sqrt{b^2 - 1} && \text{(8b)}\\
& && \text{(8c)}
\end{aligned}
$$

Manipulating the equations we should get:

$$
\begin{aligned}
a &= \frac{1}{\cos(\alpha_{min})} && \text{(9a)}\\
b &= \sqrt{\rho_{max}^2 + 1} && \text{(9b)}
\end{aligned}
$$

Now we manipulate expression 6 to obtain R_d and D_r, then we calculate in the next order:

$$
\begin{aligned}
a &= \frac{1}{\cos(\alpha_{min})} && \text{(10a)}\\
b &= \sqrt{\rho_{max}^2 + 1} && \text{(10b)}\\
R_d &= \frac{D_0}{2}\left(\frac{b+a}{b-a}\right) && \text{(10c)}\\
D_r &= D_0\left(\frac{ab}{b-a}\right) && \text{(10d)}
\end{aligned}
$$

[1] Note that this case is just a particular form of what we are going to solve at the next section

3.2. Fixing minimum angle and step size around center of the print area

Probably, your requirements on the last section give you back huge numbers for your diagonal rod and delta radius. If we want a more compact and cheap printer we should require less accuracy at the borders of printing area, but we can still ask for a particular accuracy circular area surrounding the center. First of all we should describe our x position variable in terms of distance from the center, then we'll force ρ to have a particular value ρ_{cir} at a given diameter D_{cir}.

$$D_{cir}/2 = |R_d - x| \tag{11a}$$

We now should use equation 4 and force ρ to have a given value ρ_{cir} for a given D_{cir}. This requirement is described by the next expression:

$$\rho_{cir} = \sqrt{\left(\frac{D_r}{R_d - D_{cir}/2}\right)^2 - 1} \tag{12}$$

After some manipulation and taking in care only the useful solutions of the last equation we will get the next relationship:

$$\frac{D_{cir}}{2} = \begin{cases} 0 & \forall R < \frac{D_r}{\sqrt{\rho_{cir}^2+1}} \\ R_d - \frac{D_r}{\sqrt{\rho_{cir}^2+1}} & \forall R > \frac{D_r}{\sqrt{\rho_{cir}^2+1}} \end{cases} \tag{13a}$$

We have taken only the positive part of the solution for equation 12, now you should decide what diameter D do you need for high accuracy printing area and solve the next system.

$$a = \frac{1}{\cos(\alpha_{min})} \tag{14a}$$

$$\frac{D_{cir}}{2} = R_d - \frac{D_r}{\sqrt{\rho_{cir}^2 + 1}} \tag{14b}$$

After solving we get the next equations ready for the calculation of R_d and D_r:

$$z = \cos(\alpha_{min})\sqrt{\rho_{cir}^2 + 1} \tag{15a}$$

$$R_d = \left(\frac{zD_{cir} + D_0}{z - 1}\right)\frac{1}{2} \tag{15b}$$

$$D_r = \left(\frac{D_{cir} + D_0}{z - 1}\right)\frac{z}{2\cos(\alpha_{min})} \tag{15c}$$

Make sure $z > 1$ and $R_d > \frac{D_r}{\sqrt{\rho_{cir}^2+1}}$. Smaller α_{min} means smaller R_d and same for ρ_{cir}. You may notice that for a value of $D = D_0$ we are solving the situation of the last section and for $D = 0$ you are fixing $\rho = \rho_{cir}$ at the bed center.